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RESUME OF THE THEORY OF NAVAL AND AERIAL PROPULSIVE PROPELLERS  
AND OF  
AIRPLANES IN RECTILINEAR FLIGHT.

By

A. Rateau.

Taken from l'Academie des Sciences.  
June 7, 1920.

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I have the honor to present and dedicate to the Academy the work on the theory of propulsive propellers and on the theory of airplanes which is about to appear. Though dissimilar, these two subjects have been united because they have some points in common. The computation of the movement of an airplane can only be correctly established if we are in a position to know exactly the thrust and resisting torque of the propeller for the various values of slip, which may vary greatly according to circumstances.

In the second part I have reproduced the four Papers presented to the Academy of Sciences in June and July, 1919, and have added several developments, especially on the characteristic curves of airplanes. I would call attention to two discontinuities fairly often presented by these curves, one previously noted, at large incidences greater than those used in practice; the other, on the contrary, at small incidences. These discontinuities or changes in sweep, correspond to modifications in the regime of the airflow about the wings.

The first part of the work, concerning propellers (naval and aerial) is almost entirely new. Founded on the hypotheses and ideas which I expounded in my Papers read before the Academy

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on February 19th and March 12th, 1900, the present theory is pushed much further. It leads to formulas and consequences which appear to correspond very closely to actual facts, as is clearly shown by the comparison between the theoretic results and those of the best experiments published on models of naval propellers, (D. W. TAYLOR and R. E. FROUDE) and on models of aerial propellers, (W. F. DURAND).

From the beginning I introduce as a fundamental variable, the true slip  $\delta$  with respect to the effective pitch, which is the advance per revolution of the propeller corresponding to no thrust, and I divide the section of the cylinder of fluid attacked by the propeller into two principal zones: one central, of total action, the other annular of partial action. The distinction between these two zones is based on the coefficient of influence  $k$  the value of which, in the vicinity of 2, seems to be greater for air than for water. With aerial propellers of the usual form, the action is entirely partial. But this way of taking things is not exactly in accordance with the facts of the case. To be more accurate we should add two other zones, one between the two just mentioned, the other, due to marginal effect. This subject is extremely complicated, and we shall only be able to examine it thoroughly when we are in the possession of very precise and reliable test data.

My theory leads to the following conclusions:

1st. - The thrust of a propeller is given in kilograms by the relation

$$F = b \omega n^2 (\delta - e' \delta^2) \quad (1)$$

- $\omega$  being the specific weight of the fluid in which the propeller is acting, in  $\text{kg:m}^3$ .
- $n$  the number of revolutions per second,
- $\delta$  the true slip, with respect to the effective pitch,
- $e'$  a coefficient, generally small, increasing with  $p$ , the ratio of effective pitch to diameter of a value nearly

$\frac{p^2}{2.5 p^2 + \pi^2}$  in which  $\pi$  is the ratio of circumference to diameter.

2nd. - The resisting torque of the propeller, in kilogrammeters, is given by

$$\sqrt{\quad} = \frac{bH}{2\pi} \omega n^2 (a + \delta - e \delta^2) \quad (2)$$

- $H$  being the effective pitch in meters,
- $e$  a coefficient near  $0.5 + e'$ ,
- $a$ , a quantity equal to  $\frac{4\epsilon}{(\sin 2\alpha)^2}$ ,  $\epsilon$  being the coefficient of slowing down of the order of 0.005, but varying according to the propeller,
- $\alpha$  the angle of inclination of the direction of the air leaving the blades of mean radius  $R$  (defined in the work) to the perpendicular plane of the axis.

$\epsilon$  is minimum, and  $a$  also, for a slip near to that of  $\delta_m$  which gives maximum efficiency;  $\epsilon$  and  $a$  increase from the minimum about proportionally to  $(\delta - \delta_m)^2$

3rd. - The efficiency of the propeller has for simplified expression

$$\rho = \frac{1 - \delta}{1 - \frac{\delta}{2} + \frac{a}{\delta}} \quad (3)$$

4th. - Except in the case of very narrow blades, the factor of thrust  $b$  is of a compound nature on account of the co-existence of two regimes of functioning. Its general expression is

$$b = C \frac{H^2}{g} S_1 + \frac{3 k \pi}{g} H R S_2 \quad (4)$$

$S_1$  being the area of the circle, the radius  $R_1$  limiting the zone of action (total) in square meters (not counting the hub);  $RS_2$  the moment, with respect to the center of the propeller, of the surface projected, on a plane perpendicular to the axis, of the parts of the blades outside the circle  $R_1$ .

$k$  the coefficient of influence provisionally estimated at 1.54 for water and 1.95 to 2.37 for air.

$g$  the constant of gravity in meters per second.

$C$  a coefficient equal to  $1 - \frac{p_1^2}{2} \text{ nat. log. } 1 + \frac{\pi^2}{p_1^2}$ ,  $p_1$

being the ratio of pitch to the diameter  $2R_1$  of the circle limiting the total zone of action.

The factor  $b$  does not depend on the coefficient  $\epsilon$  of the loss of relative speed, and it is the same for the thrust  $F$  of the propeller, except for the slight correction given by a more complete formula.

5th. - If there is no zone of total action, as is the case with the usual two-blade aerial propellers, the expression is simplified; the first term disappears, and there remains:

$$b = \frac{2 k \pi}{g} H R S = \frac{k \pi^2}{4g} \rho \mu \Sigma D^4$$

- $\mu$  being the ratio of the radius of the center of the moments of the surfaces of the blades projected with respect to the center of the circle of rotation to the rim radius.
- $\Sigma$  the ratio of the surface of the projected blades to the surface of the peripheral circle.
- $D$  the peripheral diameter in meters.

6th. - Provided the propeller is not too irregular, our general formulas can be very conveniently applied over the whole field of possible variations of  $\sigma$ : from 0 to 1; but they adapt themselves more exactly within the limits of slips occurring in practice: from 0.1 to 0.5; the differences between these and R. E. FROUDE'S experimental curves, which seem to result from very exact measurements, do not exceed a few thousandths.

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